FURTHER RESULTS ON INTUITIONISTIC FUZZY *B*-ALGEBRAS

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ABSTRACT. *In the present paper, further results on intuitionistic fuzzy B-algebras are studied, and some new examples are constructed. Firstly, several properties of intuitionistic fuzzy subalgebras of B-algebras are investigated. Furthermore, the notion of an intuitionistic fuzzy normal is defined, and related properties are investigated. Finally, it is proved that every intuitionistic fuzzy normal set in a B-algebra is an intuitionistic fuzzy B-algebra.*

Keywords: *B*-subalgebra; fuzzy *B*-algebra; intuitionistic fuzzy *B*-algebra.

1. INTRODUCTION

BCK-algebras and *BCI*-algebras are two classes of abstract algebras introduced by Imai and Iséki [1,2]. It is known that the class BCK-algebras is a proper subclass of the class *BCI*-algebras. In [3,4], Neggers and Kim introduced a class of algebras, called *B*-algebra, which is related to several classes of algebras of interest such as *BCK/BCI/BCH*-algebras. In [5], Kim and Yeom introduced the notion of Quotient B-algebras via fuzzy normal B-algebras. Park and Kim [6] studied quadratic B- algebras. Also, Saeid [7] initiated the concept of interval-valued fuzzy B-algebras and investigated many properties. In [8], Yoon and Kim studied structure of *B*-algebras for *B*-homomorphisms and characterized interesting properties. Other related concepts on *B*-algebras are given in [9,10]. In [1], Zadeh introduced the concept of a fuzzy set, later on, the generalization of the notion of the fuzzy set has been studied and investigated by several researchers. As a follow-up, the idea of an "intuitionistic fuzzy set" is first introduced by Atanassov [12]. In [8], Jun et al. introduced the notion of *B*-algebras and characterised many properties. Also, several related notions based on fuzzy set theory in different al- algebras, are given in [13,14,15,16,7,18,19]. Some published papers, connected to the present work, are listed below.

- In [20], Jun et al. applied the notion of fuzzy sets to B-algebras and introduced the notion of fuzzy B-algebras.
- Neggers and Kim studied a fundamental theorem of B-homomorphism for B-algebras in [4].
- In [21], Saeid initiated the notion of fuzzy topological B-algebras and studied related properties.
- In 2011, Senapati et al. represented the concept of fuzzy closed ideals of B-algebras [22] while in 2012, they also gave the notion of fuzzy B-subalgebras of B-algebra with respect to t-norm [23].

Motivated by a lot of work in this direction, in this paper, as a generalization of fuzzy *B*-algebra, we discuss intuitionistic fuzzy theory applied to *B*-algebras. We introduce the notion of intuitionistic fuzzy *B*-algebras, and investigates several properties. We organize this paper as follows: In Section 2, some fundamental notions of B-algebras are presented. In Section 3, the notion of intuitionistic fuzzy *B*-algebras is defined, and related properties are investigated with many examples. In the last, we discuss the conclusions of this work with some future directions.

2. Preliminaries

A *B-algebra* is a nonempty set F with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(b1) $\varpi * \varpi = 0$

(b2) $\varpi * 0 = \varpi$

(b3) $(\varpi * \rho) * \upsilon = \varpi * (\upsilon * (0 * \rho))$

for all $\varpi, \rho, \upsilon \in F$. A nonempty subset N of a B-algebra F is called a *B-subalgebra* of F if $\varpi * \rho \in N$ for any $\varpi * \rho \in N$. A nonempty subset N of a B-algebra F is said to be *normal* if $(\varpi * a) * (\rho * b) \in N$ whenever $\varpi * \rho \in N$ and $a * b \in N$. A nonempty subset N of a B-algebra F is said to be *nonzero normal* if $(\varpi * a) * (\rho * b) \in N$ whenever $\varpi * \rho \in N$ and $a * b \in N$ for all $\varpi * \rho \in F$ and $a(\neq 0), b(\neq 0) \in F$. Note that any normal subset N of a B-algebra F is a B-subalgebra of F, but the converse is not true (see [4], Proposition 3.4 and Example 3.5]). A nonempty subset *N* of a *B*-algebra *F* is called a (*nonzero*) *normal B-subalgebra* of *F* if it is a *B*-subalgebra which is (nonzero) normal.

Lemma 2.1. [3] *If F is a B-algebra, then* $\varpi * \rho = \varpi * (0 * (0 * \rho))$ *for all* $\varpi * \rho \in F$.

Let F be a nonempty set. A map $\Omega : F \to [0,1]$ is called a fuzzy set in F, and the complement of a fuzzy set Ω in F, denoted by $\bar{\Omega}$, is the fuzzy set in F given by

$$\bar{\Omega}(\varpi) = 1 - \Omega(\varpi) \text{ for all } \varpi \in F.$$

An intuitionistic fuzzy set (IFS, for short) $\tilde{X}$ in a nonempty set F is an object having the form

$$\widetilde{X} = \{(\Omega, \Omega_{\widetilde{X}}(\varpi), (\varphi, \varphi_{\widetilde{X}}(\varpi)) : \varpi \in F\}$$

where the functions $\Omega_{\widetilde{X}} : F \to [0,1]$ and $\varphi_{\widetilde{X}} : F \to [0,1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$0 \le (\Omega_{\widetilde{X}}(\varpi) + \varphi_{\widetilde{X}}(\varpi)) \le 1, \forall \varpi \in F.$$

An IFS $\widetilde{X} = \{(\Omega, \Omega_{\widetilde{X}}(\varpi), (\varphi, \varphi_{\widetilde{X}}(\varpi)) : \varpi \in F\}$ in F can be identified to an ordered pair $(\Omega_{\widetilde{X}}, (\varphi_{\widetilde{X}})$ in $I^F \times I^F$ For the sake of simplicity, we shall use the symbol $\widetilde{X} = (\Omega_{\widetilde{X}}, (\varphi_{\widetilde{X}})$ for the IFS $\widetilde{X} = \{(\Omega, \Omega_{\widetilde{X}}(\varpi), (\varphi, \varphi_{\widetilde{X}}(\varpi)) : \varpi \in F\}$

Clearly, every fuzzy set Ω in F is an IFS of the form $(\Omega, \overline{\Omega})$.

Definition 2.2.

Let $\widetilde{X} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ and $\widetilde{Y} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ be IFSs in F. Then

(1) $\widetilde{X} \subseteq \widetilde{Y}$ *iff* $\Omega_{\widetilde{X}}(\varpi) \le \Omega_{\widetilde{Y}}(\varpi)$ and $\varphi_{\widetilde{X}}(\varpi) \le \varphi_{\widetilde{Y}}(\varpi)$ for all $\varpi \in F$,

(2) $\widetilde{X} = \widetilde{Y}$ *iff* $\widetilde{X} \subseteq \widetilde{Y}$ and $\widetilde{Y} \subseteq \widetilde{X}$,

(3) $\overline{\widetilde{X}} = (\varphi_{\widetilde{X}}, \Omega_{\widetilde{X}})$,

(4) $\widetilde{X} \cap \widetilde{Y} = (\Omega_{\widetilde{X}} \vee \Omega_{\widetilde{Y}}, \varphi_{\widetilde{X}} \wedge \varphi_{\widetilde{Y}})$,

(5) $\widetilde{X} \cup \widetilde{Y} = (\Omega_{\widetilde{X}} \wedge \Omega_{\widetilde{Y}}, \varphi_{\widetilde{X}} \vee \varphi_{\widetilde{Y}})$,

(6) $\Delta\widetilde{X} = (\Omega_{\widetilde{X}}, \overline{\Omega}_{\widetilde{X}})$,

(7) $\Diamond\widetilde{X} = (\overline{\varphi}_{\widetilde{X}}, \varphi_{\widetilde{X}})$,

One can generalize the operations of intersection and union in Definition 2.2 to arbitrary family of IFSs as follows:

Definition 2.3. (C¸oker [24]) Let $\widetilde{X} : i \in J$ be an arbitrary family of IFSs in F. Then

- $\cap \widetilde{X}_i = \{(\varpi, \wedge\Omega_{\widetilde{X}_i}(\varpi), \vee\varphi_{\widetilde{X}_i}(\varpi)) : \varpi \in F,$
- $\cup \widetilde{X}_i = \{(\varpi, \vee\Omega_{\widetilde{X}_i}(\varpi), \wedge\varphi_{\widetilde{X}_i}(\varpi)) : \varpi \in F,$

3. Intuitionistic fuzzy *B*-algebras

In what follows, let F denote a B-algebra unless otherwise specified.

Definition 3.1. [25] An IFS $\widetilde{X} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ in F is called an intuitionistic fuzzy B-algebra if it satisfies the inequalities

$$\Omega_{\widetilde{X}}(\varpi * \rho) \ge \min\{\Omega_{\widetilde{X}}(\varpi), \Omega_{\widetilde{X}}(\rho)\} \text{ and}$$

$$\varphi_{\widetilde{X}}(\varpi * \rho) \le \max\{\varphi_{\widetilde{X}}(\varpi), \varphi_{\widetilde{X}}(\rho)\}$$

for all $\varpi \in F$.

Example 3.2. Let $F = \{0, \alpha, \beta, \gamma, \delta, \hbar\}$ be a set with the following table:

$*$	0	α	β	γ	δ	$\hbar$
0	0	β	α	γ	δ	$\hbar$
α	α	0	β	δ	$\hbar$	γ
β	β	α	0	$\hbar$	γ	δ
γ	γ	δ	$\hbar$	0	β	α
δ	δ	$\hbar$	γ	α	0	β
$\hbar$	$\hbar$	γ	δ	β	α	0

Then (F, $*$, 0) is a B-algebra (see [4], Example 3.5). Define an IFS $\widetilde{X} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ in F by

$\Omega_{\widetilde{X}}(0) = \Omega_{\widetilde{X}}(\gamma) = 0.7 > 0.1 = \Omega_{\widetilde{X}}(\varpi)$ and $\varphi_{\widetilde{X}}(0) = \varphi_{\widetilde{X}}(\gamma) = 0.2 < 0.5 = \varphi_{\widetilde{X}}(\varpi)$ for all $\varpi \in F \setminus \{0,3\}$. Then $\widetilde{X} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ is an intuitionistic fuzzy B-algebra.

Proposition 3.3. Every intuitionistic fuzzy B-algebra $\widetilde{X} = (\Omega_{\widetilde{X}}, \varphi_{\widetilde{X}})$ in F satisfies the inequalities

$\Omega_{\widetilde{X}}(0) \ge \Omega_{\widetilde{Y}}(\varpi)$ and $\varphi_{\widetilde{X}}(0) \le \varphi_{\widetilde{Y}}(\varpi)$ for all $\varpi \in F$.

Proof. Since $\varpi * \varpi = 0$ for all $\varpi \in F$, we have

$$\Omega_{\widetilde{X}}(0) = \Omega_{\widetilde{X}}(\varpi * \varpi) \ge \min\{\Omega_{\widetilde{X}}(\varpi), \Omega_{\widetilde{X}}(\varpi)\} = \Omega_{\widetilde{X}}(\varpi)$$

and

$$\varphi_{\widetilde{X}}(0) = \varphi_{\widetilde{X}}(\varpi * \varpi) \le \max\{\varphi_{\widetilde{X}}(\varpi), \varphi_{\widetilde{X}}(\varpi)\} = \varphi_{\widetilde{X}}(\varpi)$$

for all $\varpi \in F$.

For any elements ϖ and ρ of F, let us write $\varpi^n * \rho$ for $\varpi * (\cdots * (\varpi * (\varpi * \rho)))$ where ϖ occurs n times.

Proposition 3.4. Let an IFS $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ in F be an intuitionistic fuzzy B-algebra and let $n\in N$. Then, for all $\varpi\in F$,

(i) $\Omega_{\tilde{X}}(\varpi^{n}*\varpi)\geq\Omega_{\tilde{X}}(\varpi)$ and

$\varphi_{\tilde{X}}(\varpi^{n}*\varpi)\leq\varphi_{\tilde{X}}(\varpi)$ where n is odd,

(ii) $\Omega_{\tilde{X}}(\varpi^{n}*\varpi)=\Omega_{\tilde{X}}(\varpi)$ and

$\varphi_{\tilde{X}}(\varpi^{n}*\varpi)=\varphi_{\tilde{X}}(\varpi)$ where n is even.

Proof. Let $\varpi\in F$ and assume that n is odd. Then $n=2k-1$ for some positive integer k. Observe that

$\Omega_{\tilde{X}}(\varpi^{n}*\varpi)=\Omega_{\tilde{X}}(0)\geq\Omega_{\tilde{X}}(\varpi)$ and

$\varphi_{\tilde{X}}(\varpi^{n}*\varpi)=\varphi_{\tilde{X}}(0)\leq\varphi_{\tilde{X}}(\varpi)$ for all $\varpi\in F$.

Suppose that $\Omega_{\tilde{X}}(\varpi^{2k-1}*\varpi)\geq\Omega_{\tilde{X}}(\varpi)$ and

$\varphi_{\tilde{X}}(\varpi^{2k-1}*\varpi)\leq\varphi_{\tilde{X}}(\varpi)$ for a positive integer k.

Then

$$\Omega_{\tilde{X}}(\varpi^{2(k+1)-1}*\varpi)=\Omega_{\tilde{X}}(\varpi^{2k+1}\varpi*\varpi)$$

$$=\Omega_{\tilde{X}}(\varpi^{2k-1}*\varpi*(\varpi*\varpi)))$$

$$=\Omega_{\tilde{X}}(\varpi^{2k-1}*(\varpi*0)) \quad \text{by (b1)}$$

$$=\Omega_{\tilde{X}}(\varpi^{2k-1}*\varpi) \quad \text{by (b2)}$$

$$\geq\Omega_{\tilde{X}}(\varpi)$$

and

$$\varphi_{\tilde{X}}(\varpi^{2(k+1)-1}*\varpi)=\varphi_{\tilde{X}}(\varpi^{2k+1}\varpi*\varpi)$$

$$=\varphi_{\tilde{X}}(\varpi^{2k+1}*(\varpi*(\varpi*\varpi)))$$

$$=\varphi_{\tilde{X}}(\varpi^{2k-1}*(\varpi*0)) \quad \text{by (b1)}$$

$$=\varphi_{\tilde{X}}(\varpi^{2k-1}*\varpi) \quad \text{by (b2)}$$

$$\leq\varphi_{\tilde{X}}(\varpi),$$

which proves (i). Similarly we have the second part.

Proposition 3.5. If an IFS $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ in F is an intuitionistic fuzzy B-algebra, then

(fB1) $\Omega_{\tilde{X}}(0*\varpi)\geq\Omega_{\tilde{X}}(\varpi)$,

$\varphi_{\tilde{X}}(0*\varpi)\leq\varphi_{\tilde{X}}(\varpi),\ \forall\varpi\in F$,

(fB2)

$\Omega_{\tilde{X}}(\varpi*(0*\rho))\geq\min\{\Omega_{\tilde{X}}(\varpi),\Omega_{\tilde{X}}(\rho)\}$, and

$\varphi_{\tilde{X}}(\varpi*(0*\rho))\leq\max\{\varphi_{\tilde{X}}(\varpi),\varphi_{\tilde{X}}(\rho)\},\forall\varpi\in F$

Proof. For any, we have $\varpi,\rho\in F$

$$\Omega_{\tilde{X}}(0*\varpi))\geq\min\{\Omega_{\tilde{X}}(0),\Omega_{\tilde{X}}(\varpi)\}=\Omega_{\tilde{X}}(\varpi),$$

$$\varphi_{\tilde{X}}(0*\varpi)\leq\max\{\varphi_{\tilde{X}}(0),\varphi_{\tilde{X}}(\varpi)\}=\varphi_{\tilde{X}}(\varpi),$$

$$\Omega_{\tilde{X}}(\varpi*(0*\rho))\geq\min\{\Omega_{\tilde{X}}(\varpi),\Omega_{\tilde{X}}(0*\rho)\}$$

$$\geq\min\{\Omega_{\tilde{X}}(\varpi),\Omega_{\tilde{X}}(\rho)\},$$

and

$$\varphi_{\tilde{X}}(\varpi*(0*\rho))\leq\max\{\varphi_{\tilde{X}}(\varpi),\varphi_{\tilde{X}}(0*\rho)\}$$

$$\leq\max\{\varphi_{\tilde{X}}(\varpi),\varphi_{\tilde{X}}(\rho)\},$$

proving the results.

Since $\varpi=0*(0*\varpi)$ (see [9], Lemma 3.5), if $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ is an intuitionistic fuzzy B-algebra, then

$$\Omega_{\tilde{X}}(\varpi)=\Omega_{\tilde{X}}(0*(0*\varpi))\geq\min\{\Omega_{\tilde{X}}(0),\Omega_{\tilde{X}}(0*\varpi)\}$$

$$=\Omega_{\tilde{X}}(0*\varpi)$$

and

Hence $\Omega_{\tilde{X}}(\varpi)=\Omega_{\tilde{X}}(0*\varpi)$ and

$\varphi_{\tilde{X}}(\varpi)=\varphi_{\tilde{X}}(0*\varpi)$ for any $\varpi\in F$.

Theorem 3.6. If an IFS $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ in F satisfies (fB1) and (fB2), then $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ is an intuitionistic fuzzy B-algebra.

Proof. Assume that an IFS $\widetilde{X}=(\Omega_{\tilde{X}},\varphi_{\tilde{X}})$ in F satisfies the conditions (fB1) and (fB2) and let $\varpi,\rho\in F$. Using Lemma 2.1, (fB1) and (fB2), we have

$$\Omega_{\tilde{X}}(\varpi*\rho)=\Omega_{\tilde{X}}(\varpi*(0*(0*\rho)))$$

$$\geq\min\{\Omega_{\tilde{X}}(\varpi),\Omega_{\tilde{X}}(0*\rho)\}$$

$$\geq\min\{\Omega_{\tilde{X}}(\varpi),\Omega_{\tilde{X}}(\rho)\},$$

and

$$\varphi_{\tilde{X}}(\varpi*\rho)=\varphi_{\tilde{X}}(\varpi*(0*(0*\rho)))$$

$$\leq\max\{\varphi_{\tilde{X}}(\varpi),\varphi_{\tilde{X}}(0*\rho)\}$$

$$\leq \max\{\varphi_{\tilde{X}}(\varpi), \varphi_{\tilde{X}}(\rho)\}.$$

Hence $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ is an intuitionistic fuzzy *B*-algebra.

Definition 3.7. An IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ in *F* is said to be intuitionistic fuzzy normal if it satisfies the inequalities

$$\Omega_{\tilde{X}}(\varpi * a) * (\rho * b)) \geq \min\{\Omega_{\tilde{X}}(\varpi * \rho), \Omega_{\tilde{X}}(a * b)\}$$

and

$$\varphi_{\tilde{X}}(\varpi * a) * (\rho * b)) \leq \max\{\varphi_{\tilde{X}}(\varpi * \rho), \varphi_{\tilde{X}}(a * b)\}$$

for all $a, b, \varpi, \rho \in F$. If the elements *a* and *b* are nonzero, then we say that $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ is intuitionistic fuzzy nonzero normal

Example 3.8. If we define an IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ by

$\Omega_{\tilde{X}}(0) = \Omega_{\tilde{X}}(\alpha) = \Omega_{\tilde{X}}(\beta) = 0.8,$

$\Omega_{\tilde{X}}(\gamma) = \Omega_{\tilde{X}}(\delta) = \Omega_{\tilde{X}}(\hbar) = 0.3,$

$\varphi_{\tilde{X}}(0) = \varphi_{\tilde{X}}(\alpha) = \varphi_{\tilde{X}}(\beta) = 0.1,$ and

$\varphi_{\tilde{X}}(\gamma) = \varphi_{\tilde{X}}(\delta) = \varphi_{\tilde{X}}(\hbar) = 0.6$ in Example 3.2, then $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ is an intuitionistic fuzzy normal set in *F*.

Example 3.2. Let $F = \{0, \alpha, \beta, \gamma\}$ be a set with the following table:

$*$	0	α	β	γ
0	0	γ	β	α
α	α	0	γ	β
β	β	α	0	γ
γ	γ	β	α	0

Then (*F*, ∗, 0) is a B-algebra (see [20]). If we define an IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ in *F* by $\Omega_{\tilde{X}}(0) = 0.7,$ $\Omega_{\tilde{X}}(\beta) = 0.5,$ $\Omega_{\tilde{X}}(\alpha) = \Omega_{\tilde{X}}(\gamma) = 0.3,$ $\varphi_{\tilde{X}}(0) = 0.2, \varphi_{\tilde{X}}(\beta) = 0.4$ and $\varphi_{\tilde{X}}(\alpha) = \varphi_{\tilde{X}}(\gamma) = 0.6,$ Then $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ is an intuitionistic fuzzy normal set in *F*.

Theorem 3.10. Every intuitionistic fuzzy normal set $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ in *F* is an intuitionistic fuzzy *B*-algebra.

Proof. For any $\varpi, \rho \in F$, we have

$$\begin{aligned}\Omega_{\tilde{X}}(\varpi * \rho) &= \Omega_{\tilde{X}}((\varpi * \rho) * (0 * 0)) \\ &\geq \min\{\Omega_{\tilde{X}}(\varpi * 0), \Omega_{\tilde{X}}(\rho * 0)\} \\ &= \min\{\Omega_{\tilde{X}}(\varpi), \Omega_{\tilde{X}}(\rho)\},\end{aligned}$$

and

$$\begin{aligned}\varphi_{\tilde{X}}(\varpi * \rho) &= \varphi_{\tilde{X}}((\varpi * \rho) * (\rho * 0)) \\ &\leq \max\{\varphi_{\tilde{X}}(\varpi * 0), \varphi_{\tilde{X}}(\rho * 0)\} \\ &= \max\{\varphi_{\tilde{X}}(\varpi), \varphi_{\tilde{X}}(\rho)\}.\end{aligned}$$

Hence $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ is an intuitionistic fuzzy *B*-algebra.

The converse of Theorem 3.10 is not true. For example, the intuitionistic fuzzy *B*-algebra $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ in Example 3.2 is not intuitionistic fuzzy normal, since

$$\begin{aligned}\Omega_{\tilde{X}}((\beta * \hbar) * (\delta * \alpha)) = \Omega_{\tilde{X}}(\beta) &< \Omega_{\tilde{X}}(\gamma) \\ &= \min\{\Omega_{\tilde{X}}(\beta * \gamma), \Omega_{\tilde{X}}(\hbar * \alpha)\}.\end{aligned}$$

Definition 3.11. An IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ in *F* is called an intuitionistic fuzzy (nonzero) normal *B*-algebra if it is an intuitionistic fuzzy *B*-algebra which is intuitionistic fuzzy (nonzero) normal.

Example 3.12. The IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$ discussed in Examples 3.8 and 3.9 is indeed an intuitionistic fuzzy normal *B*-algebra.

Proposition 3.13. If an IFS $\tilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$) in *F* is an intuitionistic fuzzy normal *B*-algebra, then $\Omega_{\tilde{X}}(\varpi * \rho) = \Omega_{\tilde{X}}(\rho * \varpi)$ and $\varphi_{\tilde{X}}(\varpi * \rho) = \varphi_{\tilde{X}}(\rho * \varpi)$ for all $\varpi, \rho \in F$.

Proof. Let $\varpi, \rho \in F$. Then

$$\begin{aligned}\Omega_{\tilde{X}}(\varpi * \rho) &= \Omega_{\tilde{X}}((\varpi * \rho) * 0) \\ &= \Omega_{\tilde{X}}((\varpi * \rho) * (\varpi * \varpi)) \\ &\geq \min\{\Omega_{\tilde{X}}(\varpi * \varpi), \Omega_{\tilde{X}}(\rho * \varpi)\} \\ &= \Omega_{\tilde{X}}(\rho * \varpi),\end{aligned}$$

and

$$\begin{aligned}\varphi_{\tilde{X}}(\varpi * \rho) &= \varphi_{\tilde{X}}((\varpi * \rho) * 0) \\ &= \varphi_{\tilde{X}}((\varpi * \rho) * (\varpi * \varpi))\end{aligned}$$

$$\leq \max\{\varphi_{\tilde{X}}(\varpi * \varpi), \varphi_{\tilde{X}}(\rho * \varpi)\}$$
$$= \max\{\varphi_{\tilde{X}}(0), \varphi_{\tilde{X}}(\rho * \varpi)\}$$
$$= \varphi_{\tilde{X}}(\rho * \varpi).$$

Interchanging ϖ with ρ we obtain $\Omega_{\tilde{X}}(\rho * \varpi) \geq \Omega_{\tilde{X}}(\varpi * \rho)$ and $\varphi_{\tilde{X}}(\rho * \varpi) \geq \varphi_{\tilde{X}}(\varpi * \rho),$ which proves the proposition.

Theorem 3.14. If an IFS $\widetilde{X} = (\Omega_{\tilde{X}}, \varphi_{\tilde{X}})$) in F be an intuitionistic fuzzy normal B-algebra. Then the sets

$$F_{\Omega_{\tilde{X}}} := \{\varpi \in F : \Omega_{\tilde{X}}(\varpi) = \Omega_{\tilde{X}}(0)\}$$

and

$$F_{\varphi_{\tilde{X}}} := \{\varpi \in F : \varphi_{\tilde{X}}(\varpi) = \varphi_{\tilde{X}}(0)\}$$

are normal B-subalgebras of F.

Proof. It is sufficient to show that $F_{\Omega_{\tilde{X}}}$ and $F_{\varphi_{\tilde{X}}}$ are normal. Let $a, b, \varpi, \rho \in F$ be such that $\varpi * \rho \in F_{\Omega_{\tilde{X}}}$, $a * b \in F_{\Omega_{\tilde{X}}}$, $\varpi * \rho \in F_{\varphi_{\tilde{X}}}$, and $a * b \in F_{\varphi_{\tilde{X}}}$. Then

$$\Omega_{\tilde{X}}(\varpi * \rho) = \Omega_{\tilde{X}}(0) = \Omega_{\tilde{X}}(a * b)$$

and

$\varphi_{\tilde{X}}(\varpi * \rho) = \varphi_{\tilde{X}}(0) = \varphi_{\tilde{X}}(a * b)$. It follows that

$$\Omega_{\tilde{X}}((\varpi * a) * (\rho * b)) \geq \min\{\Omega_{\tilde{X}}(\varpi * \rho), \Omega_{\tilde{X}}(a * b)\}$$
$$= \Omega_{\tilde{X}}(0)$$

and

$$\varphi_{\tilde{X}}((\varpi * a) * (\rho * b)) \leq \max\{\varphi_{\tilde{X}}(\varpi * \rho), \varphi_{\tilde{X}}(a * b)\}$$
$$= \varphi_{\tilde{X}}(0).$$

Combining Proposition 3.3, we concludes that

$\Omega_{\tilde{X}}((\varpi * a) * (\rho * b)) = \Omega_{\tilde{X}}(0)$ and

$\varphi_{\tilde{X}}((\varpi * a) * (\rho * b)) = \varphi_{\tilde{X}}(0).$ which shows that

$(\varpi * a) * (\rho * b) \in F_{\Omega_{\tilde{X}}}$ and

$(\varpi * a) * (\rho * b) \in F_{\varphi_{\tilde{X}}}$. This concludes the proof.

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